Optimal rotation with differently-discounted benefit streams

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Abstract

The case is often now made that discount rates should decline with time. Underlying reasons include that some kinds benefit (or cost) might be discounted at a lower rate than that used for others: in particular, that rates for carbon values and environmental amenities might be less than that for timber. A lengthening sequence of rotations then arises, whether the benefits are consumptive ones realised at the rotation end, or non-consumptive ones whose annual value increases through the rotation. A timber discount rate lower than that for non-consumptive benefits leads to a shortening sequence of rotations. The results differ importantly from those of discounting at a reducing rate through time.

Keywords: differential discount rates, optimal rotation

Introduction

At the meeting of the Scandinavian Society of Forest Economics in Lom (Price, 2008), I presented a tentative model of how an optimal sequence of lengthening rotations could be calculated, for circumstances in which the discount rate declined through time. The solution protocol was rather unstable, and a more successful model was produced for the Faustmann III symposium (Price, 2011). Both papers explored reasons put forward for declining discount rates, and reviewed some criticisms of those arguments; there is a fuller critique in Price (2004).

A more convincing – though still contested – case can be made for applying different discount rates to different kinds of benefit (or cost). This too causes rotation length to change through time, since the benefits to which the lower discount rate is applied will have greater influence as later rotations unfold.

Differentiated discount rates

As an alternative to the popular illusion that *passage of time itself* is the mediator of declining rates, one could simply analyse segregated streams of benefit and cost, using differentiated discount rates – a main factor which gives rise to an apparent decline through time of overall discount rates. The focus being thus moved to the *causes* of different rates rather than the *effects*, there is less potential to draw spurious conclusions or to respond inappropriately to changing circumstances.

It has long been argued that benefits arising from the *presence* of a forest, and *increasing through with crop age*, would lengthen rotations (Hiley, 1956). Benefits in this category include landscape and recreation values, habitat for (some kinds of) flora and fauna, storage of carbon, and generation of non-timber forest products. Formal introduction of the effect into economic models of rotation is usually attributed to Hartman (1976). Those who have discussed this further include Strang (1976), Calish, Fight and Teeguarden (1978), Johansson and Löfgren (1985), and Price (1987). Krutilla and Fisher (1972) and Fisher and Krutilla (1972) have argued for the increasing importance of such values in natural resource economics generally. But the *combination* of these two effects – increasing importance of non-consumptive values, and their effect on rotation – seems to have been treated little if at all.

The model

The following demonstration of the rotational effect of differential discount rates is based on a model with the following reasonable characteristics.

- Timber has no net sale value until a given age – 20 years in this case.
- Thereafter revenue per hectare rises rapidly at first, then at a decreasing rate.
- No thinning revenues nor time-profiled costs are included. They would not be expected to change the overall shape of results presented below.
- The discounted sum of annual management costs is invariant with rotation, so does not affect optimal rotation length.
- First formation cost is common to all rotations so has no influence, and the final revenue is treated as being net of regeneration costs.
- Discount rates used range down from 3.5%, the rate advocated by the UK Treasury for 0-30 years, down to 1%, advocated for beyond 300 years.

The solution algorithm is slightly modified from that used for declining discount rates.

- o Initially, rotations are set arbitrarily long (200 years).
- o Starting from the first year of the first rotation, the effect on NPV is tested of shortening the current rotation and bringing forwards all subsequent ones. This continues through the rotation until an age is reached at which shortening the rotation would decrease NPV: this is the provisional optimal rotation.
- o The process is repeated, with the second rotation being provisionally optimised in the same manner. Increasing, as this does, the NPV of the

second rotation, it may now be advantageous further to shorten the first rotation, in order to bring forwards this increased value.

- Now the third rotation is provisionally optimised, and so on, until no further shortening of any individual rotation improves the NPV of the entire sequence of rotations.

The solution appears at first to evolve chaotically, as shortening of earlier rotations makes space to add extra rotations within the time horizon, and as change in rotations between iterations leaves missing values. But, as the earlier rotations begin to stabilise, the final pattern emerges slowly but persistently, until a stable solution is reached.

Results with two different terminal benefits

In this version of the problem, both kinds of benefit arise only at the end of the rotation. Timber is one obvious benefit of this kind: others are not so easy to conceive, but an "instant" carbon value arises if timber is used to displace high-embodied-carbon structural materials. The discount rate for timber (50% of the initially expected value) is 3.5% but that for the other benefit is only 1% – a figure adopted to display the limitations of the model. The expected result is that, as rotations succeed one another, the benefit with a low discount rate will dominate the result more and more, with rotation lengthening. Figure 1 shows this effect.

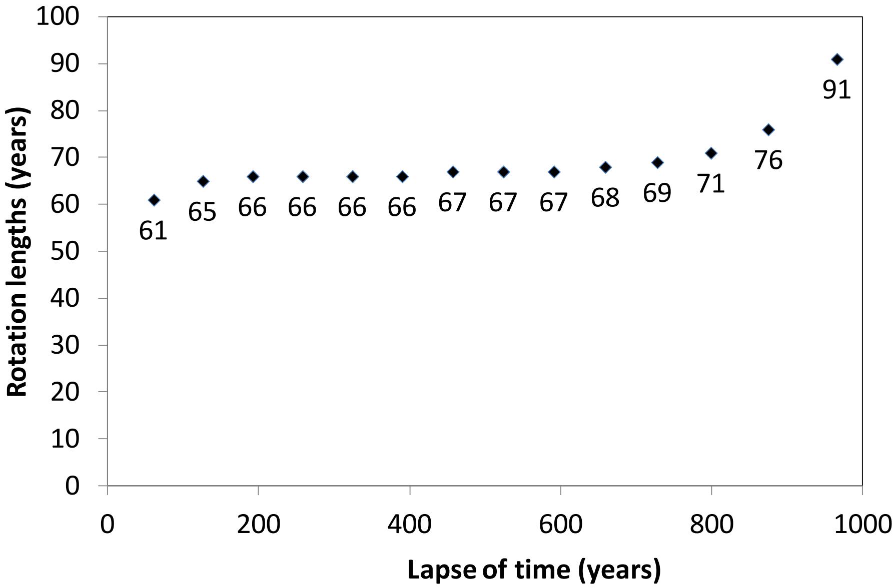

Figure 1: Lengthening rotation with two terminal benefits

An unexpected feature is that, instead of becoming asymptotic to what the optimal rotation would be if *all* benefits were discounted at 1% (66 years) the rotation eventually begins to lengthen more rapidly. This is an artefact, only significant at very low discount rates, of the limited 1000-year horizon of the simulation: as this horizon is approached, successor rotations have less and less opportunity cost, so there is increasing advantage in prolonging the rotation. A warning should be taken: clearly – and especially if some elements of benefit are discounted at a low rate – arbitrary time horizons can exert their effect long before the current rotation's NPV can be directly affected by it.

Continuous benefit

More commonly discussed than the case of two terminal benefits, is the situation in which an annual, non-consumptive benefit arises through the rotation, typically growing as the size of trees and age of stand increase. In the model used here, the *annual* benefit is taken to be proportional to what the timber growing stock's sale value would be if felled at this age. This yields the dashed line in figure 2 as the cumulative value of (undiscounted) non-consumptive benefits through the rotation. The annual value of such benefits is set at 2% of whatever the value the timber growing stock has at any time. Notice the difference between the profiles, suggesting that even without differential discounting the non-consumptive benefits could exert a considerable lengthening of rotation.

Figure 2: The values of timber and non-consumptive benefit

Considering only the benefit of timber, discounted at 3.5%, the optimal rotation would be 50 years: as expected, constant through time – the Faustmann rotation.

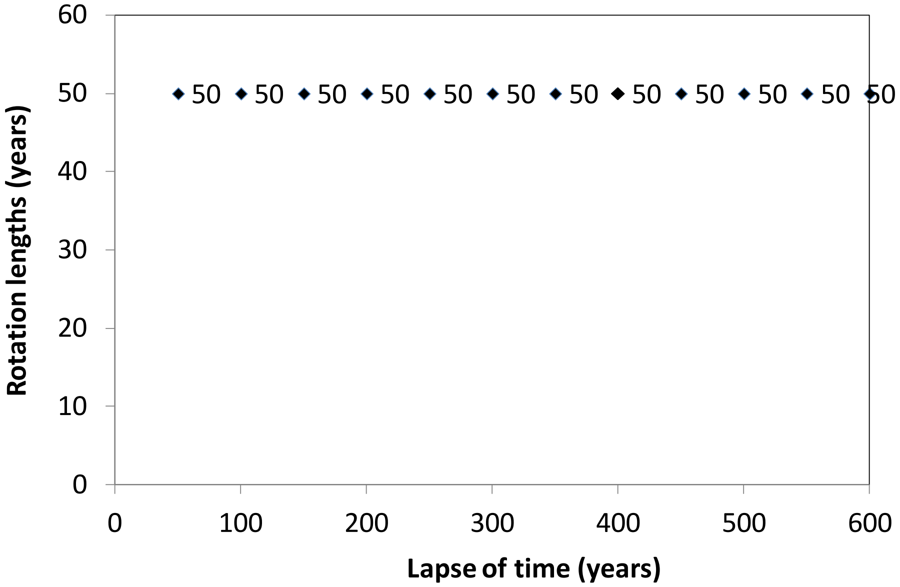

Figure 3: The result with a single discount rate, for timber only

With non-consumptive benefit initially at 1% of timber growing stock value, timber discounted at 3.5%, non-consumptive benefit discounted at 3%, and initial rotation set at 200 years, the optimal sequence of rotations is as shown in figure 4.

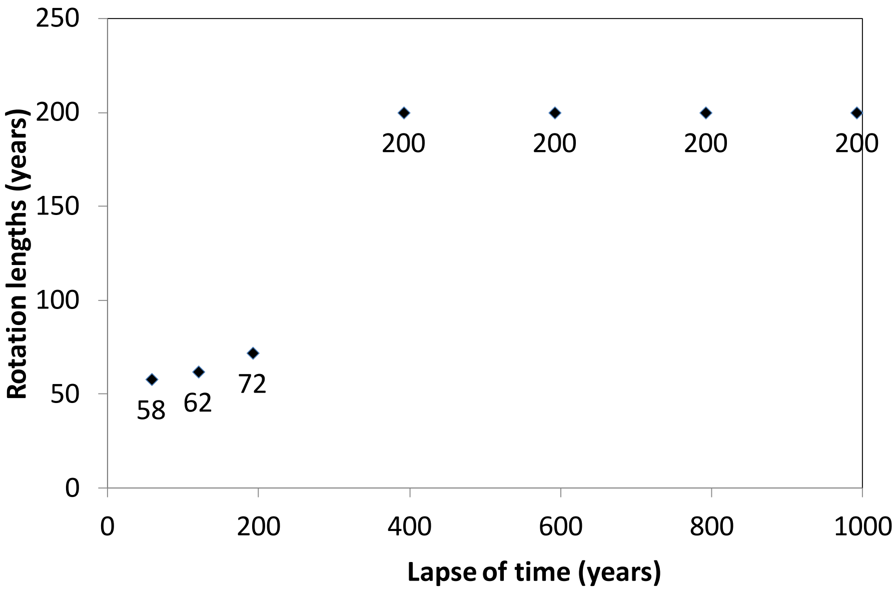

Figure 4: Lengthening rotation with timber and non-consumptive values

Note that, even with the small differential in the discount rates, eventually the optimum increases rather dramatically to the longest rotation allowed: the longer the rotation, the more influential becomes the next addition to the annual flow of non-consumptive benefits.

If the discount rate for non-consumptive benefit is 2%, or if annual non-consumptive benefit is initially as high as 2% of growing stock value, there is no transition of rotation: the optimum from the outset is, in effect, the growth span of the tree or even longer. (As the selected revenue function increases monotonically through time, this speculation cannot be checked.)

There are also forest influences which are increasingly negative through the rotation, and may grow increasingly important through time, for example losses of water for hydroelectricity generation (Barrow et al., 1986; Price, 1999). Their effect too can be modelled, with the expected result: rotations shorten progressively. Figure 5 is based on an initial annual cost 0.5% of growing stock value, discounted at 3%, as against the 3.5% timber discount rate. If conditions prevail into the distant future, the optimal rotation eventually converges on the longest one that has no adverse effect, 20 years in this case.

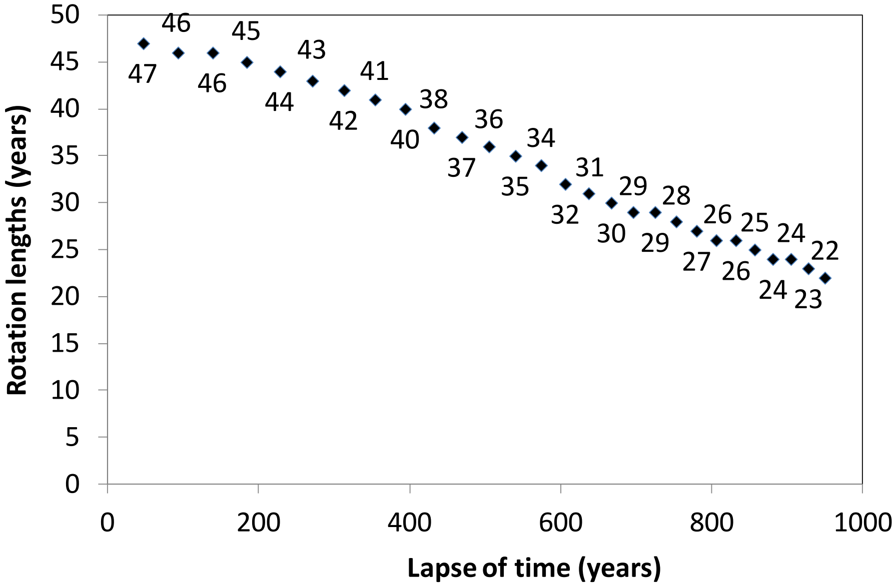

Figure 5: Shortening rotation with lower discount rate for annual *costs*

One could also envisage increasing timber shortage relative to provision of the public goods of non-consumptive environmental effects. Discounting timber revenues at 2.5% and water losses at 3%, the optimal rotation lengthens from 52 to 54 years (which is the optimal timber rotation at 2.5%).

With annual non-consumptive benefit initially at 1% of growing stock value, timber discounted at 3%, and non-consumptive benefit at 3.5%, the rotation shortens to 52 years, which is what the optimum would be, if there were no annual benefits.

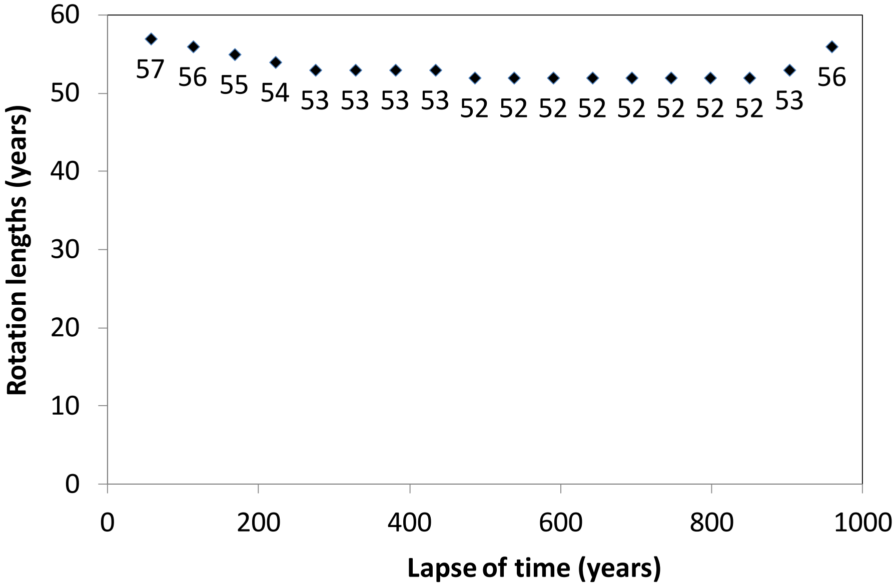

Figure 6: Timber discounted at a lower rate than non-consumptive values

And if annual non-consumptive benefits are initially 10% of timber benefits, the decline is more dramatic, essentially to the same asymptote.

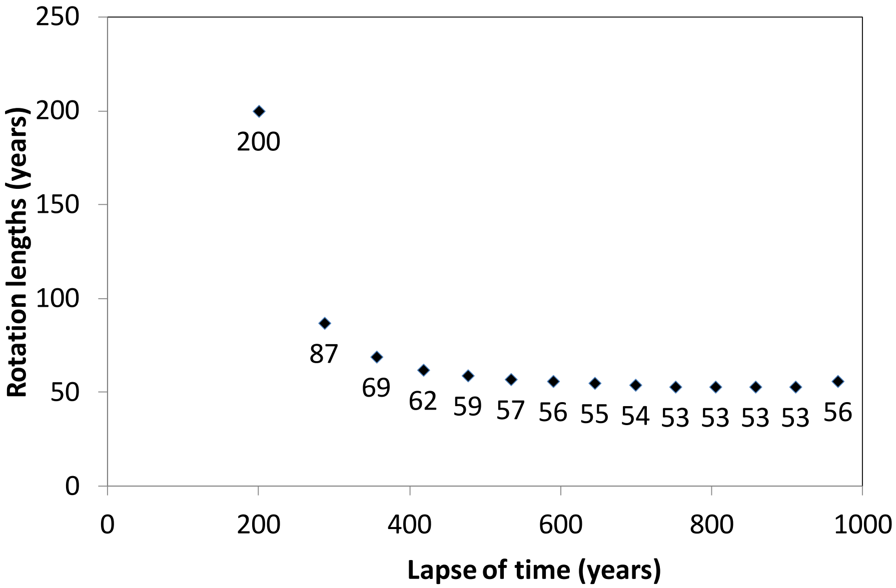

Figure 7: Timber discounted at a lower rate than initially high non-consumptive values

Conclusions

So, are there any important conclusions, except the obvious one: that a low discount rate for one element of crop value will tend to change rotation lengths progressively? Perhaps so.

- The effect is *not* the same as that of a discount rate declining through time, which should therefore not be used as a surrogate for discount rates differentiated by class of benefit.
- There is a major difference of effect between benefits arising at the time of timber harvest, and those non-consumptive *benefits* which increase in relative value with both passing time and size of tree. The former's rotation tends to an asymptotic value, at the optimal rotation for whichever benefit attracts the lowest discount rate. The latter by contrast lengthens progressively and ultimately becomes indefinitely protracted, whatever the starting conditions. But conditions which increase the relative importance of timber revenues lead to an optimal rotation which is asymptotic through time to the optimal timber rotation.
- Eventually – if the factors causing the differential of discount rate persist – the rotation may be prolonged indefinitely. This indeed has happened already in some forests and some stands in many forests where non-consumptive values dominate. "Long-term retentions" represent a management category which recognises this factor. Differential discount rates simply reflect a slow shift in forest values that will bring a greater proportion of forest area into this category. Consequent reductions in timber supply should bring about its own premia on timber, which may balance or even reverse the indicated trends, to follow a rotation pattern as shown in figure 6.
- While the path of values in the very long term is of course hard to predict, the examples above suggest that this should not be a major problem for forest managers. If non-market benefits are already sufficiently important, they will already determine the optimal rotation. If they are only expected to become important in the rather distant future, there is no need to deviate much from the present rotation for timber. The indicated progressive lengthening of rotation is slow at first, and adjustments can be made as the prospect over the coming rotation seems to indicate.
- Shifts in rotation, if semi-permanent, do bring problems for maintaining normal age-class structure. For those stands which have little environmental value, short-term shortfalls of timber production may be compensated by somewhat early final felling, together with thinning of the retained stands where this is compatible with their non-consumptive use.

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